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ABSTRACT

Presented are transcriptions of a research panel presentation held at the annual meeting of the National Council of Teachers of Mathematics, Boston, Massachusetts, April 1979. The members of the panel are all involved in working directly with children in a clinical setting for pupils with learning disabilities in mathematics and with graduate students in courses on diagnosis and treatment of learning disabilities in mathematics. This document is divided into sections on Clinical Intervention Research as follows: (1) introduction, (2) development of diagnostic and assessment procedures, (3) instructional procedures and exemplars, (4) a clinical case study, (5) mathematics education curriculum specifications, and (6) summary and conclusions. (MP)

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SOME FINDINGS OF AND FUTURES FOR CLINICAL RESEARCH IN MATHEMATICS EDUCATION

An Introduction to Clinical Intervention Research

John W. Wilson

Application of Clinical Intervention Research: Diagnosis

Barbara R. Sadowski

Application of Clinical Intervention Research: Instructional Procedures and Exemplars

Carolynn Washbon

Application of Clinical Intervention Research: A Clinical Case Study

Martin L. Johnson

Application of Clinical Intervention Research: Curriculum Specifications

John W. Wilson

Summary

Robert Ashlock

These papers are transcriptions of a panel discussion held at the Annual Meeting of the National Council of Teachers of Mathematics, Boston, April 1979. Additional information concerning the content of the papers may be obtained by writing the panel organizer, Dr. Martin L. Johnson, Acting Director, The Arithmetic Center, University of Maryland, College Park, MD 20742

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An Introduction to Clinical Intervention Research

John W. Wilson

What brings the panel members together is that each of us has been heavily involved in working directly with children in a clinic for children with learning disabilities in mathematics and with the graduate students in our courses on diagnosis and treatment of learning disabilities in mathematics. However, the focus today is not on the service to children nor the training aspects of professionals in our respective clinic programs. Our presentations today are about a type of research that we have been conducting as an integral part of the service and training aspects of our clinic programs. Each of us on the panel, separately, collectively, and with our graduate students have for several years now been involved in a kind of research we have come to refer to as "Clinical Intervention Research". I will first give a brief overview of 1) what we mean by that kind of research, and 2) the characteristics that distinguish it from other types such as experimental and correlational research. Then each of the panel members will present, describe and illustrate some of the kinds of problems and techniques using clinical intervention procedures that they have been involved in. They will be working on specific themes: some on instructional procedures, some on diagnostic procedures and some on curriculum specification procedures.

What then are some of the characteristics of clinical intervention research? Perhaps we can best get a picture of it by contrasting it with some of the better known and most frequently reported research such as experimental, correlational and factor analytic. Clinical intervention research is quite clearly distinguishable from those on several clearly easily observable characteristics such as the type of data collected. Experimental research tends to emphasize the collection of

measurable data, whereas in clinical intervention research we are interested primarily in the kinds of thought processes that can be reported on and on the detailed descriptions of the task we are involved in, e.g., anecdotal records and case study kinds of procedures. The methods reflecting the data are different from experimental and correlational research. The methods of analyzing the data are also quite different. The selection of the children or the subjects for our research are quite different. Obviously in experimental research, as you know, we go out and select subjects and try to get them into various groups for experimental control purposes. You also have a particular plan in mind for testing hypotheses in experimental research. Children we work with in clinical intervention research studies come to us in a serendipitous manner. They are referred to the clinic because of the difficulties that they are having in learning mathematics. As we work with them on a one-to-one basis, or sometimes in small groups, we are conducting what we define as clinical intervention research.

The nature of the treatments or interventions in clinical intervention research are quite different from the fixed specified treatments that occur in experimental designs. Clinical intervention research is also pursued in quite a different milieu. Some of the details of these characteristics which distinguish clinical intervention research from experimental research will be given by other members of the panel as they present their particular approaches to the problems they have dealt with.

However, in our view the major difference, i.e., the major characteristic which distinguishes clinical intervention research from experimental is the <u>purpose</u> for which this research is conducted. The purpose of experimental studies is essentially to confirm or reject one or more hypotheses by developing tightly

controlled designs which eliminate as far as possible all the rival hypotheses concerning the phenomenon being studied. And, of course, this "confirmation" process usually uses some inferential statistic to assign a degree of probability, not certainty of course, but probability that the hypotheses of interest does account for the phenomenon observed.

The major purpose of clinical intervention research on the other hand is to generate, to create, or to uncover hypotheses, not to test or confirm them. It is also the intention of clincial intervention research to generate these hypotheses in a manner which gives these hypotheses the necessary a priori probability for later confirmation to be of use in experimental designs using inferential statistics. Hence, clinical intervention research is generative research, while experimental research is primarily confirmative. There are other types of generative research besides clinical intervention research. We see experimental, correlational, theoretical and the clinical intervention studies as phases of a total research program. The type of total program that Tyler speaks of in some of his work. "Research is not a monolithic enterprise. It does not involve a single kind of person. It is a collective enterprise of considerable magnitude involving a variety of talents and leading to a variety of activity in order to be maximally effective." (Tyler, 1967, p. 140).

So if the purpose of clinical intervention research is to generate hypotheses, what kind of hypotheses have we been generating? The hypotheses we have been particularly interested in generating tend to fall into broad categories or themes. There are those that deal with improving the instructional procedures; all aspects of it, the kinds of materials, the sequence of instruction, the type of teacher

moves involved in developing the various kinds of learning with children. And the interaction of these instructional procedures with learner organismic variables is another area of interest for generating hypotheses. That's one strand, instructional procedures.

Another category of hypotheses that we have been seeking to generate deals with the development and improvement of procedures for assessing learning in mathematics -- especially diagnostically valid and precise assessment procedures. Dr. Sadowski will be speaking on that theme. Dr. Washbon will be speaking on the instructional procedure theme.

A third category of hypotheses we are seeking deals with what we refer to as a curriculum specification problem; that is, determining specifics such as what kinds of objectives and how best to state the objectives of instruction. We have been questioning for some time the use of behavioral objectives and exploring other ways we might express the objectives of instruction. We are concerned with which meanings for a given topic, or which conceptualization of a given topic is best for what stage of learning of a child. There are different meanings for each of the operations, for example. Which of these seem to be best for improving children's learning? We are interested in the question of sequencing of content, what kinds of concepts preceed or are subsumed by other concepts and what kinds of sequences seem the best to facilitate the child's progress through this system we call mathematics. I will be presenting some of the information on that as well as Martin Johnson. In my case I will be talking about some work we have been doing on the meanings of the operations, and in problem solving. Dr. Johnson will be illustrating the kind of intensive clinical intervention study with

one child that has led to some of the hypotheses we have then gone on to work with on other kinds of research. Within each of these themes or categories we have to assume not only the generation of specific and testable hypotheses but we have sought to develop theoretical models that guide our search for the patterns that then can be translated into hypotheses. We have sought to develop models which also help us to critique and better design other kinds of research. You will hear some of the speakers refer to the development of some guiding models along this line.

For the instructional procedures questions we have something called the activity type cycle. We have another one that students love to roll off their tongues, the exemplar matrix. We also have other models for helping us think through what assessment procedures are needed. We talk about behavioral indicators and a cross-class translation matrix. Each of these models have been described in some detail in other publications (Wilson, 1976). Another model that helps us think through the kinds of objectives and how we might organize them and also guides us in our thinking about organizing curricular issues is referred to as the content taxonomy. We use what we prefer calling content objectives rather than behavioral objectives. We are trying to organize our content objectives into something we call content structures rather than behavioral hierarchies. So our approach is then quite different as reflected by our guiding models.

Within each of the themes, also some of us have gone beyond generating specific hypotheses and developing guiding models and moved to the next stage of designing and conducting various types of experimental and/or correlational studies to test the hypotheses generated in our clinical intervention research.

Therefore, we want to convey our belief that the use of clinical intervention research is not being suggested by this panel as a replacement for experimental research or a replacement for the other kinds of research, but is a phase in a cyclic development of an ongoing collective research effort.

Application of Clinical Intervention Research: Diagnosis

Barbara R. Sadowski

My talk is based on research involved with the development of diagnostic procedures for the identification of strengths and weaknesses in arithmetic. The model which has been the most helpful in the development of this research is the content taxonomy developed by John Wilson (1976). The content taxonomy structures the content of elementary school mathematics first along the lines of general sets such as whole numbers, nonnegative rationals, integers and then rationals. Each of these sets is further subdivided into areas involving concepts about (1) the numbers which are elements of the set, (2) meanings of the operations with those numbers, (3) properties of the operations, (4) principles of and skill with computation, (5) sentence solving, (6) verbal problem solving and (7) measurement. A category could then be addition computation with whole numbers and as such would have a specific code in the content taxonomy, in this case IV D 1.

The content taxonomy by its comprehensiveness has been helpful in the development of assessment procedures. One assessment procedure is a paper and pencil diagnostic test and another is an interview protocol which involves the use of structured manipulative materials. Because of the limited time this morning, I am going to address myself mainly to the development of the diagnostic arithmetic test. The content taxonomy is helpful in test construction by serving as a comprehensive outline for a table of specifications. The organization of the content taxonomy also alerts us to possible relationships between the concepts in the various categories delineated by the taxonomy. For example, we could hypothesize that a relationship exists between the concepts which underlie the

meanings of the operations and skill in sentence solving and verbal problem solving. This relationship is investigated as part of the research being discussed this morning and is reported elsewhere (Sadowski, 1978).

One of the benefits of using the content taxonomy is that it is possible to be more precise in the diagnosis of a child's arithmetic difficulties. For example, if I say to you that a child has trouble with addition, and you are now going to design some instructional procedures which will remediate this difficulty, you might ask, "What do you mean by trouble with addition? Do you mean the child has trouble with the algorithm, or trouble with sentences involving addition, or trouble understanding the meaning of addition or even that the child does not recognize addition when presented in a verbal problem?" If, however, a code from the content taxonomy is included as part of the diagnosis of the child, the diagnosis becomes much more precise and allows us to communicate rather effectively with a minimum amount of verbiage. In fact, most of us go around communicating in shorthand that the child has trouble in IV A 2a, IV D 1-2 which translates into meaning that he cannot do addition and subtraction computation and does not understand concepts about the notational system for whole numbers. At this time, this behavior is unique to students of the University of Maryland program.

The content taxonomy was first used to help in the analysis of commercially available diagnostic arithmetic tests. Each item on a test was examined and after it was decided what concept the test constructors were testing for, the item was placed in one of the content taxonomy categories. We found out that some diagnostic tests covered certain categories very well, e.g., notational system concepts. In other categories there were only a few items or no items

at all. Because of the inadequate coverage of arithmetic content on these commercially available tests, in 1975 we constructed a diagnostic test which was developed using the content taxonomy as the table of specifications. This test is called the MDAT or Maryland Diagnostic Arithmetic Test. Currently, categories covered by the MDAT are whole numbers and non-negative rationals.

The test items on the original MDAT were used with the children who are referred to the Arithmetic Clinic for diagnosis. The diagnosis is conducted in a one-on-one interview with the child. The MDAT is not a timed, standardized, norm-referenced test which yields a grade equivalent score. It consists of items categorized by content category. After the child works these items an interview follows about his/her work on the test. From these interviews, notes are taken about what the child does correctly, what he does incorrectly, and the processes he uses in arriving at answers. After using the MDAT in this form for one semester we found that there were some items on the MDAT that just didn't seem to provide much information, while others seemed to act as flags for the child's lack of concepts. If a child missed one of these items, you could almost say that he was weak in that entire category. Of course there were exceptions to this, but generally it seemed to hold true. Based on the results of the interviews with the children from the clinic, we revised many of the items, discarded some and added others. We expanded the categories in which we needed more than one item to act as a flag. For example, one item on the notational system read "7 tens + 15 ones = _____." The child was supposed to fill in the blank. Most of the children responded 715; very few answered 85. The children who did the item incorrectly seemed to focus only on the face value of the digits in the

stem. Many of the children also wrote 63 as the answer to the problem "6 tens + 3 ones" while others answered it incorrectly. Looking at the children who answered these two questions incorrectly provided us with valuable diagnostic information. This information was helpful in focusing questions during the interview with each child.

After the test revision we found that the test worked better, resulting in a more precise diagnosis. The diagnosis tended to be confirmed by the fact that children were often referred to the clinic for remediation and an intervention study was carried out with each of these children. The remediation was successful and thus the diagnosis tended to be valid.

Now it became a question of: If we can do so well with one child, can we do as well with a group of children? It is not very efficient to sit down and diagnose every single child one-to-one. So again we expanded the test, restructuring the items into domains of homogenous items. We found a statistic which is based on a latent trait model based on the assumption that two extreme types of children exist in the population with respect to the concepts being tested. There are those children who have mastered the concepts covered by a domain of items and those children who have not mastered it. In addition, the model allows for what are called intrusion and ommission parameters. In more general terms these are referred to as guessing and forgetting parameters. (See Dayton and Macready, 1976, for further details.)

The revised MDAT was given to 500 children and the domains were tested for homogenity to see whether children could be separated into two groups for each domain. Results from the group-administered MDAT indicated that about

half of the domains were homogenous. It also demonstrated that it is possible to get a diagnosis on an individual child in terms of identifying content strengths and weaknesses. The test results of the sentence solving part of the MDAT identified some interesting error patterns which led to the generation of hypotheses about the strategies which children use to solve sentences. This area of research has identified many factors which influence the difficulty of the sentences, but the interaction of these factors makes it difficult to separate the effect of each factor. The hypotheses generated by the results of this test are presently being investigated by a graduate student at the University of Houston. She is first administering the sentence solving test, then following this with a one-on-one interview to provide evidence which will confirm/disconfirm the hypotheses generated by the first test administration.

Thus, the cyclical nature of clinical intervention research is apparent in the development of assessment procedures for diagnosis of learning disabilities in arithmetic. Informal observations made during the one-on-one testing led to the development of a valid group-administered diagnostic arithmetic test, the results of which led to further hypotheses about children's processes in sentence solving, hypotheses which may or may not be confirmed by the results of interviews with children. The process continues.

Application of Clinical Intervention Research: Instructional Procedures and Exemplars

Carolyn Washbon

Dr. Wilson has referred to some of the models we have used to guide our work with children. One benefit we have derived from using guiding models is that instruction can follow quite directly from a diagnosis. Dr. Sadowski has referred to the use of the content taxonomy in the design of a diagnostic test. This model is also used in our work to specify instructional objectives. As we develop or select instructional procedures for individual children we can draw directly from diagnostic data collected about a child's concepts using the same model. In addition, as diagnoses become more precise in identifying processes as well as concepts, prescriptive instruction has the potential for becoming more effective.

The model which we refer to as the Activity Type Cycle guides both diagnosis and specification of instructional procedures. Essentially the Activity Type Cycle is based on the notion of cognitive reorganization (Brownell and Hendrikson, 1950). The cycle consists of six types of teaching/learning activities: 1) initiating, 2) abstracting, 3) schematizing, 4) consolidating, 5) transferring and 6) diagnosing. Each activity may be defined by characterizing both the critical tasks for the teacher and the purpose of the activity for the learner (See Wilson, 1976). The Activity Type Cycle is outlined in Figure 1.

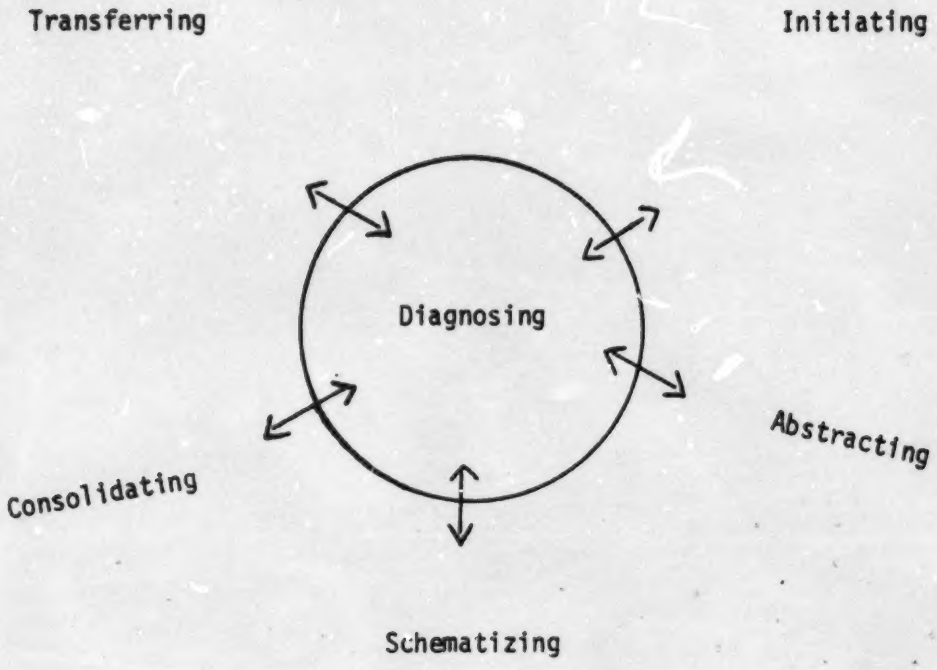

Figure 1
Activity-type Cycle

In a developmental sense we can view the cycle beginning with initiating activities, in which the child is faced with a problem (Brownell, 1942) and uses the concepts he has in order to generate instances of the new concept. These are followed by abstracting activities in which we help the child to recognize a concept in many different situations and symbolize or label the concept in convential ways. Schematizing activities help the child to organize and relate sets of concepts while consolidating activities are designed to help the child fix and practice the retrieval of those schemas he has learned. Transferring activities serve to aid the use of learned concepts in new situations and in some cases re-initiate the cycle. A good example of this re-initiating is found in using counting to solve addition situations.

Activities for diagnosing are at the center of the cycle, since these activities may be used at any time during a sequence of activities to assess the acquisition of a concept or set of concepts. Hypotheses concerning the interaction of instructional procedures with individual needs are at the heart of much of our work in clinical intervention research. They suggest directions for us in exploring Cronbach's "five-fold interactions--subject matter with type of instruction, with timing of instruction, with type of pupil with outcome." (Cronbach, 1966, p. 77).

As we have used the Activity Type Cycle to select instructional procedures for children in the clinic we have found that some topics and some children require both a greater variety and number of activities of a given activity type. Variation in children's needs might be expected, however, variations in sequences or timing of instruction lead to the hypothesis that some concepts may require specific ratios of activities that differ from other sets of concepts. The Activity Type Cycle was used to describe treatments in two experimental studies carried out in the classroom (Washbon, 1978; Whitlock, 1978). The results of these studies seem to indicate that the model may be inappropriate for use in a fixed time treatment design. A trials to criterion may be a more valid way to assess the utility of the model and allow for the variation in children's rates of learning. Research is still needed on the original hypothesis.

Another model which has guided our work is the exemplar matrix (Wilson, 1976). This model is used to evaluate materials used with children in instructional activities in terms of the sensory input from the material.

Our work with children who have perceptual or sensory handicaps has lead to the modification of some materials. One clinician working with blind children has modified a math balance to ring a bell when it balances. Another clinician found both in the clinic and the classroom that children diagnosed as having visual perception problems had difficulty using two-dimensional or pictorial exemplars; in particular those which involved a display with a space down the main diagonal in a rectangular array of square regions exemplifying the family of addition facts for 7 as shown in Figure 2.

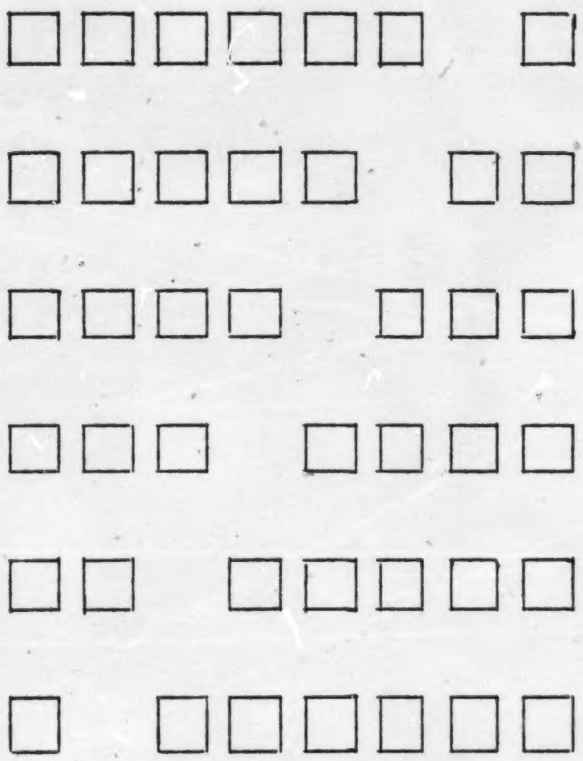

Figure 2

Some hypotheses based on these and other clinical intervention research studies are:

1) Exemplars for mathematical ideas may have relevant, irrelevant or desirable characteristics.

2) Exemplars with irrelevant characteristics may interact with the perceptual characteristics of a learner. These are only a sample of the hypotheses generated about instruction procedures and exemplar characteristics from our clinical intervention research studies.

Application of Clinical Intervention Research: A Clinical Case Study

Martin L. Johnson

What is a typical example of the type of clinical intervention research conducted in the Arithmetic Clinic? What kinds of research might be carried out by a graduate student, teacher or clinician assigned to work with a child who comes to the clinic and is diagnosed as having a particular type of learning difficulty? We will use one example to try to answer these questions. In addition what we hope to illustrate by the following example is that out of this kind of research come many questions that lead to further research.

Our example is the case of a ten year old girl who was diagnosed as having extreme difficulty in solving verbal problems. The type of problem with which she had difficulty is probably familiar to you. For example, she experienced difficulty with this problem and others like it:

> Steve had some pencils. He gave 8 to Jeff.
> Steve had 4 pencils left. How many pencils did he have to begin with?

During the initial diagnostic sessions the child was asked to choose the correct operation for a number of verbal problems and then asked to solve particular problems. One of the major difficulties the child had in performing this set of tasks was the selection of the correct operation.

The clinician was given this diagnosis, asked to explore the child's difficulty in the area of problem solving more fully, and to search the experimental literature on problem solving for an appropriate method for teaching problem solving. The literature on problem solving, as you may know, is large, diverse, and quite inconclusive in terms of "a best method". The clinician finally narrowed her search to those methods which had been found to be effective for ten year olds

having difficulty in choosing the correct operation in one step verbal problems. The treatment she selected was a variation of the "wanted-given" approach, an approach in which emphasis is placed on the meaning of the operations (Wilson, 1960). The specific procedures used with the child consisted of posing a variety of one-step verbal problems which could be translated into number sentences such as 3 + 4 = followed by a discussion of the structure of the problem. One verbatim report of a conservation between the child and the clinician went like this?

Clinician - What are you asked to do in this problem?

Child - Add.

Clinician - What numbers are given in this problem?

Child - 3 + 4

Clinician - Do you know what those numbers are called?

Child - Addends or parts.

Clinician - So now you have the addends. What do you do with them?

Child - Add them.

Clinician - When you add them what do you get?

Child - The answer, seven.

Clinician - Do you know another name for the answer?

Child - The sum or total

Clinician - So in an addition problem you are given what?

Child - Add nds

Clinician - And you are asked to find what?

Child - The sum or total.

In this procedure emphasis was placed on the meaning of an operation and making the child aware of particular relationships in verbal problems. Are you looking for an addend, a sum, a factor or a product? For example, when you are given the parts or addends and you are looking for a total or sum, you use the operation of addition to solve the problem.

In order to monitor the child's progress the clinician administered a 21 item pre-test composed of one-step verbal problems requiring addition, subtraction, multiplication or division. In addition to the overall pretest scores the clinician looked at incorrectly selected operations and incorrect computations. For example, this child often chose addition when division was appropriate. The treatment in this clinical intervention study was administered over six 15 minute periods as part of a weekly one hour session with the child at the clinic. At the end of the six sessions, the same test was given again as a post-test. As can be seen from Table 1 below, the child was able to choose the correct operation in each problem on the posttest (Hopkins, 1976, p. 52).

Table 1

Operation Required vs Operation Chosen vs Correct Computation per Operation: Pretest/Posttest

		Operation			
		Addition	Subtraction	Multiplication	Division
Pretest	OR	4	8	4	5
	CCh	4	7	1	0
	ICh	7	2	0	0
	CCh-CCo	4	6	1	0
	ICh-CCo	7	2	0	0
Pottest	OR	4	8	4	5
	CCh	4	8	3	5
	ICh	1	0	0	0
	CCh-CCo	4	7	3	5
	ICh-CCo	1	0	0	0

OR - Number of items requiring specific operation

CCh - Number of items requiring specific operation in which the operation was correctly chosen

ICh - Number of items for which this operation was incorrectly chosen.

CCH-CCo - Number of items for which the operation was correctly chosen and the computation was correct.

To find out whether the wanted-given intervention strategy would transfer to two-step verbal problems, eight problems of this form were presented.

> Dawn had 12 books. She gave 6 of those books to a friend. Then her mother gave her 4 more books. How many books does Dawn have now?

In this problem, two operations are required. First it is necessary to use subtraction, and then addition. The child was able to choose the correct operation in the two-step verbal problems in 75% of the problems and computed correctly 68% of the time. We were optimistic about this particular intervention for verbal problem solving and more rigorous testing of this instructional procedure through both clinical and experimental research designs in classrooms has been done. Thus far, it has proven to be quite effective.

We believe that the uncovering of possible hypotheses and the generation of further research questions are very important aspects of clinical intervention research. This example of a clinical intervention case study cited here gives some indication of how clinical intervention research interacts with other types of research. Many researchable hypotheses were generated from this case study.

Application of Clinical Intervention Research: Curriculum Specification

John W. Wilson

Among the things that we are looking at are: which meanings help the child, not necessarily restricting ourselves to those which are instructional procedures. In the case study Marty referred to, the investigation was not in terms of instructional procedures so much as which meanings of the operations facilitate problem solving by children. We also find that in some of the research literature there are different meanings given to augment the instructional procedures used. We need to be able to sort out the different meanings and this is what we have done in our work on the meanings of the operations. For example, in problem solving and in work on sentence solving we have been focusing on action meanings versus these things we call wanted-given meanings. There have been several experimental studies now done contrasting those. One that was done by Angela Pace (1961) some years ago and one that I did which was reported in 1967. The Arithmetic Teacher article came out of this kind of clinical investigation and others have picked up on that particular approach to meanings. So it is rather exciting to see people picking up on hypotheses that have been generated from the clinic.

Another set of meanings focuses on the meanings of division, i.e., which meanings of division you talk about. The subtractive approach or the distributive approach or the partition approach as social meanings of the mathematical meaning of division, as finding the missing factor when the product and known factor are given. The question is: Which of those meanings goes on to facilitate the development of algorithms and skills not only in division of whole numbers but on to the work in division with non-negative rationals and later on to algebraic expressions. Several of our clinical studies indicated that the distributive

procedures may be a more transferable, meaningful conceptualization of the division process than the measurement or subtractive meaning. This hypothesis has since been confirmed in a series of studies by Dilly (1970), and one of our other students, Wilmer Jones (1976). If you will look at an algebraic division then obviously the measurement procedure isn't going to function very well there. The distributive algorithm for division seems to be a stronger one for introducing to children and for expecting the meanings to carry over to work with decimals and division in algebra.

We are looking at meanings and which meanings we try to specify in our work that we expect a child to learn and we call those specifications content objectives. What we are saying is that if we want a child to have a concept, then let's define that concept. Since a concept is not an overt behavior, we are talking about content objectives as the description or definition of the concept that we want the child to have. Because we can't see a concept, we then say that for each of these content objectives there exists a constellation of behavioral indicators. We look for a broad range of behavioral indicators for inferring conceptualization by the child. So we are constantly asking what the concepts are, how can we define them, can we organize them, can they be related and then what behavioral indicators can we use to infer that the learning is going on. Therefore, the learning is something that is going on inside the child and is not an overt behavior. Overt behaviors are simply the basis for inferring the kind of learning going on inside the child. Therefore, we believe in specifying quite clearly the meanings of the operations and using these as the objectives of our instruction. Marty's presentation was emphasizing that idea, Barbara's is emphasizing the diagnosis of those meanings, i.e., which are present, which are absent, what levels of maturity the child has for these concepts; and in Carolynn's case the instructional procedures for bringing

about the optimal learning of these concepts.

I'd like to close our formal presentation, while Bob is going to pick up on the discussion and what futures of this clinical intervention research might be.

Summary and Conclusions

Robert Ashlock

Earlier this week there was a group of people who met to see if there might be interest in forming a North American section of the International Group for the Psychology of Mathematics Education. Professor Hans Freudenthal of Holland shared his conviction with that group that the nearer you are to real children in the instructional setting in your research, the more your research can be useful. There is a good deal of information that comes out of clinical intervention research, such as that described here, that can be tested and used in classroom settings. Martin Johnson gave a description of a graduate student testing out a strategy with a ten year old girl. It worked beautifully with one child, but how will it work with groups of children? John Wilson has, of course, referred to similar work that has been done in the area of problem solving.

Another of our graduate students studied partitive division with a group of ten children in her classroom (Greenbaum, 1975). She explored the relationship of division with conservation of numerousness and noted that some of the children had difficulty in explaining what they were doing in renaming 2 tens and 5 ones as 25 ones. As it turned out, the non-conservers were those who had difficulty in explaining renaming. That immediately raised questions that could be looked at in other experimental studies.

What are other areas to explore? The interaction of learner-organismic variables and instructional variables is an area that we have hardly touched. Another is the interaction of a tutor or teacher's personality and a learner's personality. Currently we are cooperating with Southern Illinois University on a study about this question.

Yet another area has to do with the dissemination of research findings. That is, we are aware that a body of research literature exists, yet as a rule classroom teachers do not work on the basis of that body of literature and its findings. If teachers are in a clinical setting as graduate students conducting clinical intervention studies in which they are surveying the literature and using it to do these studies, will it affect the way they use such literature now and in the future as opposed to the kinds of things we normally do?

One comment I would like to make about the kind of research we are doing. It requires a particular milieu. When we have tried to do this type of research without some of the elements of this milieu we have run into trouble. Required are instructors, researchers, graduate students, children, parents, materials and research literature in close proximity. This proximity makes possible both structured and informal interactions, and on a regular basis. These interactions are an integral component of clinical intervention research.

The proof that all of this works is as varied as what the panel had to say to you this morning. It is for you to judge whether we have been successful.

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